M.Sc. (MATHEMATICS)

ASSIGNMENT

Session 2024-2026 (II-Semester)

&

Session 2023-2025 (IV-Semester)

CENTRE FOR DISTANCE AND ONLINE EDUCATION

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Programme: M.Sc. (Mathematics) Semester:-IV

Important Instructions

(i) Attempt all questions from the each assignment given below. Each question carries 05 marks and the total marks are 15.

(ii) All questions are to be attempted in legible handwriting on plane white A-4 size paper and to be uploaded online to the Directorate of Distance Education for evaluation.

Nomenclature of Paper: Functional Analysis

Paper Code: MAL-641 **Total Marks = 15 + 15**

ASSIGNMENT-I

Q.1. State ad prove Riesz-Fisher Theorem.

Q.2. State and prove Open Mapping Theorem.

Q.3. Let M be a closed linear subspace of a Normed linear space$N$. If the norm of coset $x + M$ in the quotient space $\frac{N}{M}$ is defined by

$$\|x + M\| = \inf.\{\|x + m\|; m\epsilon M\}.$$

Then $\frac{N}{M}$ is a normed linear space.

ASSIGNMENT-II

Q.1. State and prove Minkowski's Inequality.

Q.2. Prove that if a normed linear space X is reflexive, then X^* is also reflexive

Q.3. State ad prove Riesz-Representation Theorem for Hilbert spaces.

Nomenclature of Paper: Differential Geometry

Paper Code: MAL-642 **Total Marks = 15 + 15**

ASSIGNMENT-I

Q.1. (a) Established **Serret Frenet formulae** $\mathbf{t}' = k\mathbf{n}$, $\mathbf{n}' = \tau\mathbf{b} - k\mathbf{t}$, $\mathbf{b}' = -\tau\,\mathbf{n}$ where the symbols have their usual meaning.

(b) For the curve $x = 3t, y = 3t^2, z = 2t^3$, show that any plane meets it in three points and deduce the equation to the osculating plane at $t = t_1$.

Q.2. **(a)** If C is a curve for which **b** varies differentially with arc length. Then to show that a necessary and sufficient condition that C is a plane curve is that $\tau = 0$ at all points.

(b) Let C be a curve given by the equation $\mathbf{r} = (u, u^2, u^3)$, find the curvature and torsion of C at the point (0,0,0). Also, find the equation of its binormal line and normal plane at the point (1,1,1).

Q.1. Given the curve $\mathbf{r} = (e^{-u}\sin u, e^{-u}\cos u, e^{-u})$. Find at any point '$u$' of this curve

(i) Unit tangent vector **t**
(ii) The equation of tangent
(iii) The equation of normal plane
(iv) The curvature
(v) The unit principal normal vector **b**, and
(vi) The equation of the binormal.

ASSIGNMENT 1I

Q.1(a) Find the envelope of the plane $3xt^2 - 3yt + z = t^3$ and show that its edge of regression is the curve of the intersection of the surfaces $y^2 = zx$, $xy = z$.

(b) Find the principal curvatures etc. on the surface generated by the binormals of a twisted curve.

Q.2.(a) Find the principal curvatures and the lines of curvature on the right helicoids $x = u\cos\phi, \quad y = u\sin\phi, \quad z = c\phi$.

(b) Find the envelope of the plane $(x/a)\cos\theta\,\sin\phi + (y/b)\sin\theta\,\sin\phi + (z/c)\cos\phi = 1$.

Q.3.(a) To prove that the envelope of a developable plane whose equation involves one parameter is a developable surface

(b) A necessary and sufficient condition that a curve on a surface be a line of curvature is that the surface normal along the curve is developable.

Nomenclature of Paper: Mechanics of Solid-II

Paper Code: MAL-643 **Total Marks = 15 + 15**

ASSIGNMENT-I

Q.1. Derive the formulae for stresses in terms of two analytic functions, assuming plane strain conditions.

Q.2. Discuss the problem of deflection of a central line of an elastic beam by transverse load.

Q.3. Derive constitutive equation for a Maxwell material. Also discuss its creep and relaxation phases.

ASSIGNMENT-II

Q.1. Find torsional moment in the problem of torsion of an elliptic cylinder.

Q.2. Obtain the frequency equation for Rayleigh waves. Also show that these are non-dispressive and particle motion is elliptic retrograde.

Q.3. Solve the problem of a long thick-walled tube in plane strain whose material is elastic in dilatation and Maxwell viscoelastic in distortion with internal pressure p and outer surface is in contact with a rigid body.

Nomenclature of Paper: Integral Equation

Paper Code: MAL-644 **Total Marks = 15 + 15**

ASSIGNMENT-I

Q.1. Find the integral equation corresponding to boundary value problem (B.V.P.)

$$y''(x) + \lambda y(x) = 0, \qquad y(0) = 0, y(1) = 1.$$

Q.2. State and prove Green's formula.

Q.3. Solve the integral equation: $y(x) = x + \lambda \int_0^{\pi} \sin(x)\, sin(t)\, y(t)dt$

ASSIGNMENT-II

Q.1. Find the resolvent kernel of Volterra Integral Equation with kernel $K(x, t) = \frac{\cosh t}{\sinh t}$.

Q.2. Transform the problem: $y''(x) + y = x,\ y(0) = 1, y'(1) = 0$ to Fredholm integral equation.

Q.3. State and prove Fredholm's Third Theorem.

Nomenclature of Paper: Advanced Fluid Mechanics

Paper Code: MAL-645 **Total Marks = 15 + 15**

ASSIGNMENT-I

Q.1. Discuss the properties of boundary layer equations.

Q.2. Obtain the equation of motion of a gas.

Q.3. Obtain the principal stresses and principal stress direction if the stress tensor at a point is given by

$$\tau_{ij} = \begin{pmatrix} 6 & 2 & 0 \\ 2 & 3 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

ASSIGNMENT-II

Q.1. Determine the local frictional coefficient for flow over a flat plate, based on Karman integral equation.

Q.2. Derive Navier-Stokes's equation of motions in Cartesian coordinates.

Q.3. Define Reynold Number, Froude number, Mach number and Eckert number.

1 Nomenclature of Paper: Computing Lab-3

Paper Code: MAP-648 Total Marks = 15 + 15

1.1 ASSIGNMENT-I

Q.1. System of equations: (5)

$$\sin nz = nz \prod_{n=1}^{\infty}\left(1 - \frac{z^2}{n^2}\right) \nabla \cdot \vec{q} = 0$$

$$u_{\frac{\partial T}{\partial x}} + v_{\frac{\partial T}{\partial y}} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + v\frac{\partial^2 T}{\partial y^2}\right)$$

Q.2 Write syntax for the following the matrix: (5)

$$\tau_{ij} = \begin{pmatrix} 6 & 2 & 0 \\ 2 & 3 & 0 \\ 0 & 0 & 4 \end{pmatrix}$$

Q.3. Discuss the commands that can be use to write multiple equations. (5)

1.2 ASSIGNMENT-II

Q.1. What is use of multiline-environment, show by an example. How IEEE eqnarray – environment is used and what are the advantages. (5)

Q.2. Syntax for the piecewise function: (5)

$$P_A(x) = \begin{cases} 1 & \text{if } x = 0 \\ 2 & \text{if } x = 1 \\ 4 & \text{if } x = -1 \end{cases}$$

Q.3. Table using `tabular`: (5)

<table>
<tr><th>X</th><th>Y</th><th colspan="2">Z</th></tr>
<tr><td rowspan="2">A</td><td>C_1</td><td>a</td><td>b</td></tr>
<tr><td>C_2</td><td>c</td><td>d</td></tr>
<tr><td rowspan="2">B</td><td>C_3</td><td>e</td><td>f</td></tr>
<tr><td></td><td>f</td><td>h</td></tr>
</table>

Programme: M.Sc. (Mathematics) Semester:-II

Important Instructions

(i) Attempt all questions from each assignment given below. Each question carries marks mentioned in brace and the total marks are 15.
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Nomenclature of Paper: Abstract Algebra

Paper Code: MAL-521 **Total Marks = 15 + 15**

Assignment-I

1. Let G be a finitely generated abelian group. Prove that G can be decomposed as a direct sum of a finite number of cyclic groups C_i, i.e.,

$$G = C_1 \oplus C_2 \oplus \cdots \oplus C_t$$

where either all C_i's are infinite or for some $j < k$, $C_1, C_2, \ldots, C_j$ are of order $m_1, m_2, \ldots, m_j$, respectively, with $m_1 \mid m_2 \mid \cdots \mid m_j$, and the rest of C_i's are infinite. (5)

2. Define nilpotent transformation with a suitable example. Also, prove that all the characteristic roots of a nilpotent transformation are zero. If $T \in A(V)$ has all its characteristic roots in F, then there exists a basis of V in which the matrix of T is triangular. (5)

3. Let V be a vector space over F and $T \in A(V)$. If the minimal polynomial of T over F is

$$f(x) = a_0 + a_1 x + \cdots + a_{m-1} x^{m-1} + x^m$$

and V is a cyclic $F[x]$-module, then prove that there exists a basis of V under which the matrix of T is the companion matrix of $f(x)$. (5)

Assignment-II

1. Let M be an R-module. Prove that the following conditions are equivalent: (5)
   (i) M is Noetherian.
   (ii) Every non-empty family of submodules of M has a maximal element.
   (iii) Every submodule of M is finitely generated.

2. Show that if R is a Noetherian ring with identity, then $R[x]$ is also a Noetherian ring. (5)

3. Define similar transformation and prove that if a subspace $W \subset V$ is invariant under T, then T induces a linear transformation $\bar{T}$ on V/W, defined by:

$$(v + W)\bar{T} = vT + W$$

   Further, if T satisfies a polynomial $q(x)$ over F, then so does $\bar{T}$. (5)

Nomenclature of Paper: Measure & Integration Theory

Paper Code: MAL-522 **Total Marks = 15 + 15**

Assignment-I

1. Prove that the characteristic function χ_A is measurable if and only if A is a measurable set. (5)

2. State and prove the **Bounded Convergence Theorem**. (5)

3. State and prove **Lusin's Theorem**. (5)

Assignment-II

1. State and prove **Egoroff's Theorem**. (5)

2. Answer the following questions:
   (i) Differentiate between Lebesgue and Riemann integration.
   (ii) What are measurable functions? Give an example.

(iii) What are convex functions?

(iv) Explain L^p-spaces with a suitable example.

(v) What are functions of bounded variation? (5)

3. Prove the following properties of integrable functions over a measurable set E:

   (i) If f, g are integrable over E, then $f + g$ is integrable and:
   $$\int_E (f+g) = \int_E f + \int_E g$$

   (ii) If $f \leq g$ almost everywhere, then:
   $$\int_E f \leq \int_E g$$

   (iii) If A and B are disjoint measurable subsets of E, then:
   $$\int_{A\cup B} f = \int_A f + \int_B f$$

   (5)

Nomenclature of Paper: Method of Applied Mathematics

Paper Code: MAL-523 **Total Marks = 15 + 15**

Assignment-I

1. Let the random variable X have the **exponential distribution** with probability density function
   $$f(x) = \lambda e^{-\lambda x}, \quad x \geq 0, \lambda > 0.$$
   Find the **mean**, **variance**, and **moment generating function (MGF)** of the distribution. (5)

2. Define the **normal distribution**. Show that the moment generating function (MGF) of a normal random variable with mean μ and variance σ^2 is
   $$M_X(t) = \exp\left(\mu t + \frac{1}{2}\sigma^2 t^2\right).$$
   Hence, deduce the **first** and **second moments** of the distribution. (5)

3. If a random variable X follows a **binomial distribution** with parameters n and p, and it is given that

$$P(X=1)=3P(X=2), \quad P(X=2)=3P(X=3),$$

find the values of n and p. Also, compute the **mean** and **variance** of the distribution. (5)

Assignment-II

1. Represent the vector

$$\vec{A}=x\hat{i}+y\hat{j}+z\hat{k}$$

in spherical coordinates (r,θ,ϕ). (5)

2. Use the method of Fourier transforms to determine the displacement $u(x,t)$ of an infinite string, given that the string is initially at rest and that the initial displacement is $f(x)$, where $-\infty < x < \infty$. (5)

3. Find the Fourier sine transform of $f(t)=e^{-at}$, where $a>0$. Hence, deduce the Fourier cosine transform of

$$f(t)=\frac{t}{a^2+t^2}.$$

(5)

Nomenclature of Paper: Ordinary Differential Equations-II

Paper Code: MAL-524 **Total Marks = 15 + 15**

Assignment-I

1. Solve the following system of linear differential equations using the **eigenvalue method**:

$$\begin{cases} \frac{dx}{dt}=4x+y \\ \frac{dy}{dt}=-2x+y \end{cases}$$

Find the general solution. (5)

2. State and prove Abel Liouville's formula. (5)

3. Determine the nature of the critical point $(0, 0)$ of the system

$$\frac{dy}{dt} = 2x - 7y, \quad \frac{dy}{dt} = 3x - 8y,$$

and determine whether or not the point is stable. (5)

Assignment-II

1. Use the calculus of variation to find the curve joining points $(0, 0, 0)$ and $(1, 2, 4)$ of shortest length. Also, find the distance between these two points. (5)

2. Analyze the following nonlinear system:

$$\begin{cases} \frac{dx}{dt} = x(1-y) \\ \frac{dy}{dt} = y(x-1) \end{cases}$$

   (a) Find the equilibrium points.

   (b) Linearize the system at each equilibrium point.

   (c) Classify the stability of each equilibrium. (5)

3. Using the **Euler–Lagrange equation**, find the function $y(x)$ that minimizes the functional:

$$J[y] = \int_0^1 \left(y'^2 + y^2\right) dx, \quad \text{with } y(0) = 0,\ y(1) = 1$$

(5)

Nomenclature of Paper: Complex Analysis-II

Paper Code: MAL-525 **Total Marks = 15 + 15**

Assignment-I

1. State and prove the Weierstrass factorization theorem. (5)

2. State and prove Jensen's formula. (5)

3. Prove that if $|z| \leq 1$ and $p \geq 0$, then

$$|1 - E_p(z)| \leq |z|^{p+1}.$$

(5)

Assignment-II

1. State and prove Hurwitz's theorem. (5)

2. State and prove the Riemann mapping theorem. (5)

3. (i) State Hadamard's factorization theorem.
   (ii) Show that

$$\sin(\pi z) = \pi z \prod_{n=1}^{\infty} \left(1 - \frac{z}{n^2}\right)$$

   by Hadamard's factorization theorem. (5)

Nomenclature of Paper: Advanced Numerical Method

Paper Code: MAL-526 **Total Marks = 15 + 15**

Assignment-I

1. Approximate the following improper integral using a suitable numerical integration method:

$$\int_1^{\infty} \frac{1}{x^2+1}\, dx$$

(5)

2. Using a second order method with $h = \frac{1}{2}$, find the solution of the BVP

$$(1+x^2)y'' + 2xy' - y = 1 + x^2, \quad y(0) = 0, \quad y'(1) = 1$$

(5)

3. The function $f(x, y)$ is known:

$$f(0,0) = -1, \quad f(0,1) = 2, \quad f(0,2) = 3, \quad f(1,0) = 4, \quad f(1,1) = 0, \quad f(1,2) = 4,$$

$$f(2,0) = 2, \quad f(2,1) = -2, \quad f(2,2) = 3$$

   For these values, construct the Newton's bivariate polynomial. Also, find the approximate values of $f(1.25, 0.75)$ and $f(1.0, 1.5)$. (5)

Assignment-II

1. Consider the first-order differential equation:

$$\frac{dy}{dx} = x + y, \quad y(0) = 1$$

   Use the 4th-order Runge-Kutta method to solve the equation numerically in the interval $x \in [0, 1]$ with step size $h = 0.1$. (5)

2. Solve the following system of linear equations using the relaxation method (also known as the Successive Over-Relaxation method) with initial guess $(x_1^{(0)}, x_2^{(0)}, x_3^{(0)}) = (0, 0, 0)$. Use a relaxation factor $\omega = 1.1$ and iterate until the absolute difference between successive values is less than 0.0001:

$$\begin{aligned} 10x_1 + 2x_2 + x_3 &= 92 \\ x_1 + 20x_2 - 2x_3 &= -44 \\ -2x_1 + 3x_2 + 10x_3 &= 22 \end{aligned}$$

   (5)

3. The following values of x and y are given:

x	1	2	3	4
y	1	2	5	11

   Find the cubic splines and evaluate $y(1.5)$ and $y'(3)$. (5)

Nomenclature of Paper: Computing Lab-Matlab

Paper Code: MAL-527 **Total Marks = 15 + 15**

Assignment-I

1. Write a program to calculate mean and median. (5)
2. Write a program to find the adjoint of a matrix. (5)
3. Write a program to operate arithmetic operators on vector. (5)

Assignment-II

1. Write a program to draw multiple graphs on same plot. (5)
2. How to solve systems of linear equations using matrices in MATLAB. (5)
3. Write a program to operate element-wise operations on matrices. (5)